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Theory of a neutron counter using proton recoils from a solid hydrogenous radiator

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With 6 figures in the text

Abstract

The sensitivity is calculated for a neutron counter using proton recoils from a solid hydrogenous radiator. The following assumptions have been made. The pulses from the chamber are proportional to the energy loss of the protons in the chamber gas. The range of the protons in the radiator is $R = R_0\,(E/E_0)^{3/2}$. The neutron-proton scattering is spherically symmetrical in the center-of-gravity system.

Theoretical calculations of neutron counters using proton recoils have been made by several authors (1), (2). A general survey is found in (3). Some calculations have already been made by CASE (1) for that special type of counter which is considered below.

The calculation has been made with the following assumptions.

The chamber is irradiated with monoenergetic neutrons perpendicular to the radiator. Proton recoils are produced in the radiator. The protons lose some part of their energy in the radiator and then give an ionization in the gas, which is proportional to the energy loss in the gas. The range-energy relation in the radiator is

$$R = R_0 \left(\frac{E}{E_0}\right)^{3/2}.$$

The neutron-proton scattering is assumed to be spherically symmetrical in the center-of-gravity system, which is correct for neutron energies less than 10 MeV.

The energy of a recoil is determined from the principles of conservation of energy and momentum.

$$E = \frac{4\, m_n m}{(m_n + m)^2} \cdot E_n \cdot \cos^2 \theta,$$

where m_n and m are the masses of the neutron and the recoil particle. E_n is the neutron energy. θ is the angle between the lines of flight of the incoming neutron and the recoil particle.

In a hydrogenous radiator $m = m_n$ and

$$E = E_n \cdot \cos^2 \theta,$$

and the maximum proton energy is equal to the neutron energy.

It is easy to show that the recoil protons will be equally distributed over the energies from 0 to E_n, if the scattering of neutrons by protons is spherically symmetrical.

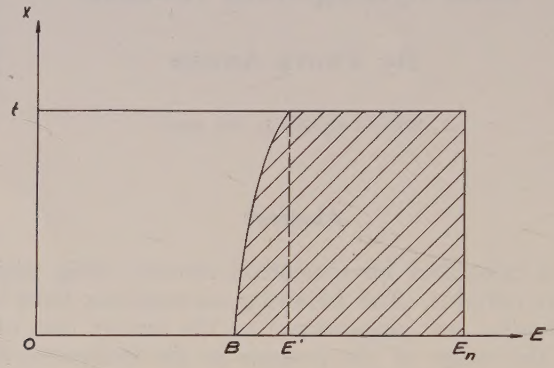

Fig. 1. Diagram for calculation of the sensitivity.

The integral sensitivity of the chamber is called

$$A = A\ (E_n,\ B,\ t),$$

where E_n = neutron energy
B = bias energy, determined by amplitude discriminator or amplitude analyser
t = thickness of the radiator.

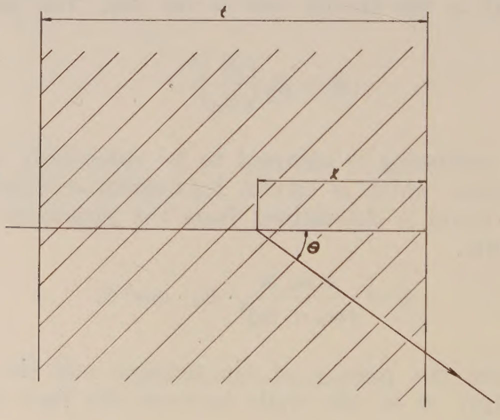

Fig. 2. Diagram for calculation of the energy loss in the radiator.

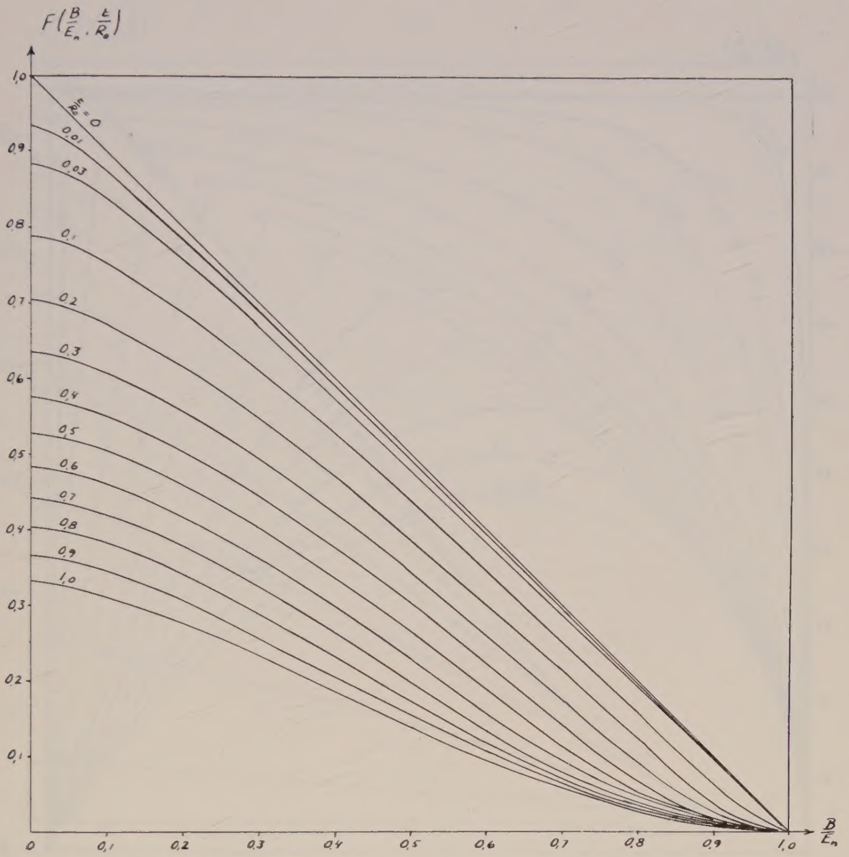

Fig. 3. Integral pulse-height distribution.

The sensitivity is defined as the ratio between the number of proton recoils, giving a pulse $> B$, and the number of neutrons passing through the radiator. The differential sensitivity a is defined as

$$a = a\,(E_n, B, t) = -\frac{\partial A}{\partial B}.$$

Fig. 1 shows a coordinate system with the recoil energy E as abscissa and the position x in the radiator, where the recoil is produced, as ordinate, x being measured from the gas side of the radiator. The distribution of primary recoil protons being constant over energy and position gives that the probability of finding a recoil in the element $dxdE$ is independent of the position of the element. To get the integral sensitivity of the chamber we have only to integrate over the area shaded in Fig. 1.

$$A\,(E_n, B, t) = N\,\sigma\,(E_n) \int\!\!\int \frac{dxdE}{E_n}.$$

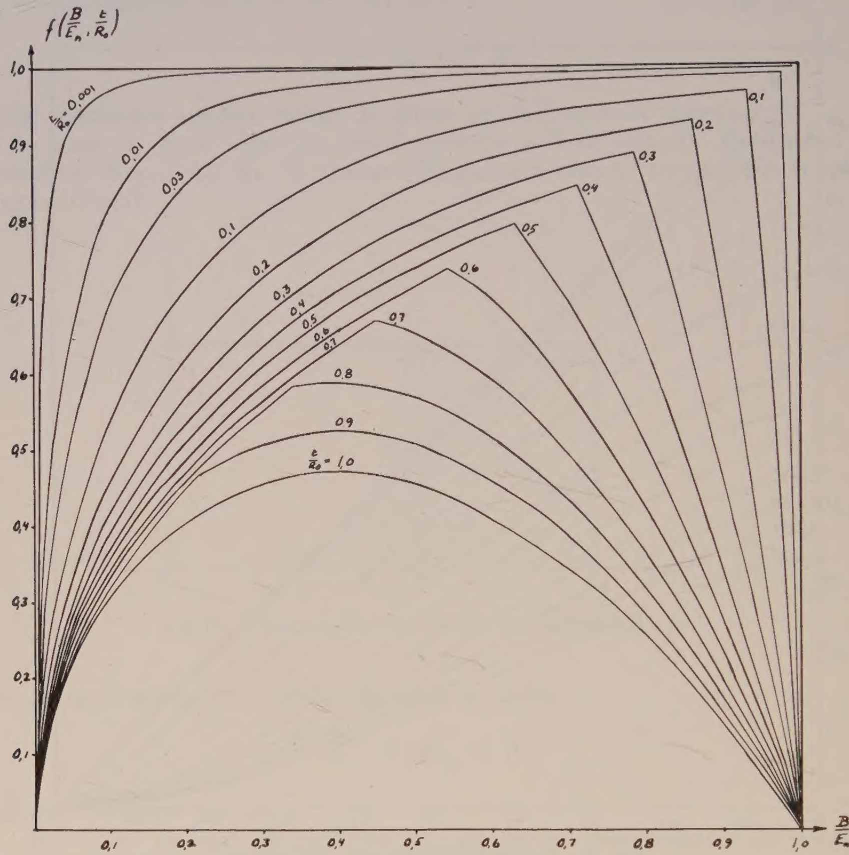

Fig. 4. Differential pulse-height distribution.

$\sigma(E_n)$ is the cross-section for neutron-proton scattering, and N is the number of hydrogen nuclei per volume unit of the radiator.

The left limit of the integration area follows from the absorption law (Figs. 1 and 2).

The path of the recoil proton is divided into two parts, one in the radiator and one in the gas

$$\frac{x}{\cos\theta} + R_0\left(\frac{B}{E_n}\right)^{3/2} = R_0\left(\frac{E_n\cos^2\theta}{E_n}\right)^{3/2}.$$

With $E = E_n \cdot \cos^2\theta$ we get

$$\frac{x}{R_0} = \left(\frac{E}{E_n}\right)^2 - \left(\frac{B}{E_n}\right)^{3/2}\left(\frac{E}{E_n}\right)^{1/2}$$

which is the equation for the left integration limit in Fig. 1.

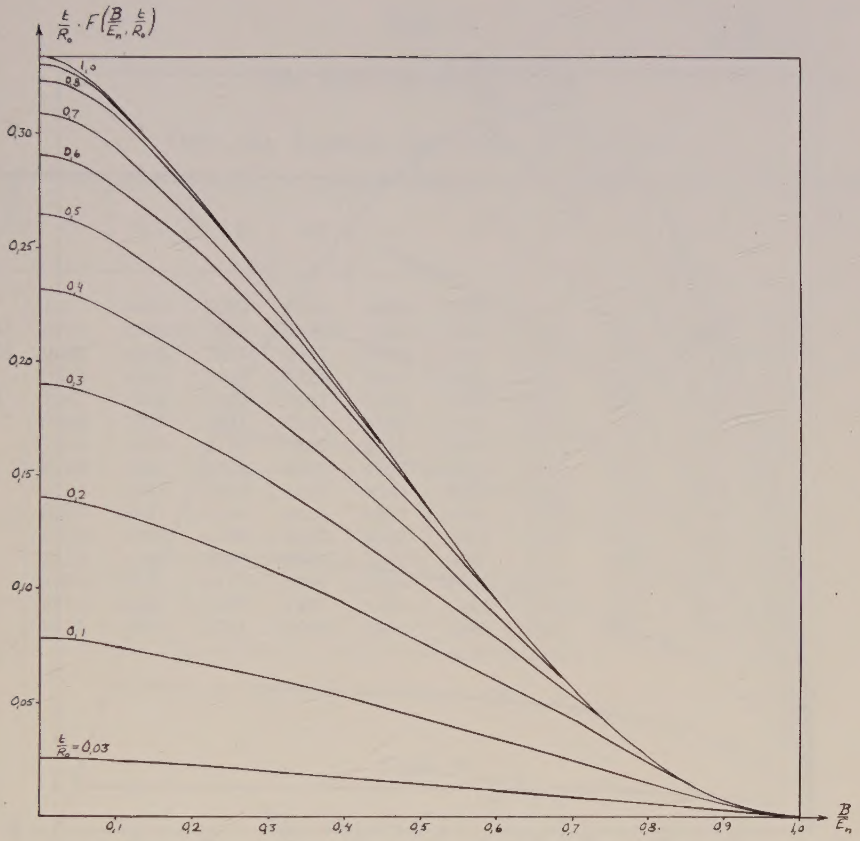

Fig. 5. Integral pulse-height distribution multiplied by the thickness of the radiator.

The computation will be different for the following two cases.

I) The left limit cuts the line $x = t$ in the point (E', t), where E' is found from

$$\frac{t}{R_0} = \left(\frac{E'}{E_n}\right)^2 - \left(\frac{B}{E_n}\right)^{3/2} \left(\frac{E'}{E_n}\right)^{1/2}.$$

II) The left limit cuts the line $E = E_n$.

The integration must be done with the help of the parameter E'.

$$\text{I)} \qquad \frac{A}{N \sigma R_0} = \frac{1}{R_0} \int_B^{E'} x(E) \cdot \frac{dE}{E_n} + \frac{1}{R_0} \int_{E'}^{E_n} t \cdot \frac{dE}{E_n},$$

$$\text{II)} \qquad \frac{A}{N \sigma R_0} = \frac{1}{R_0} \int_B^{E_n} x(E) \cdot \frac{dE}{E_n}.$$

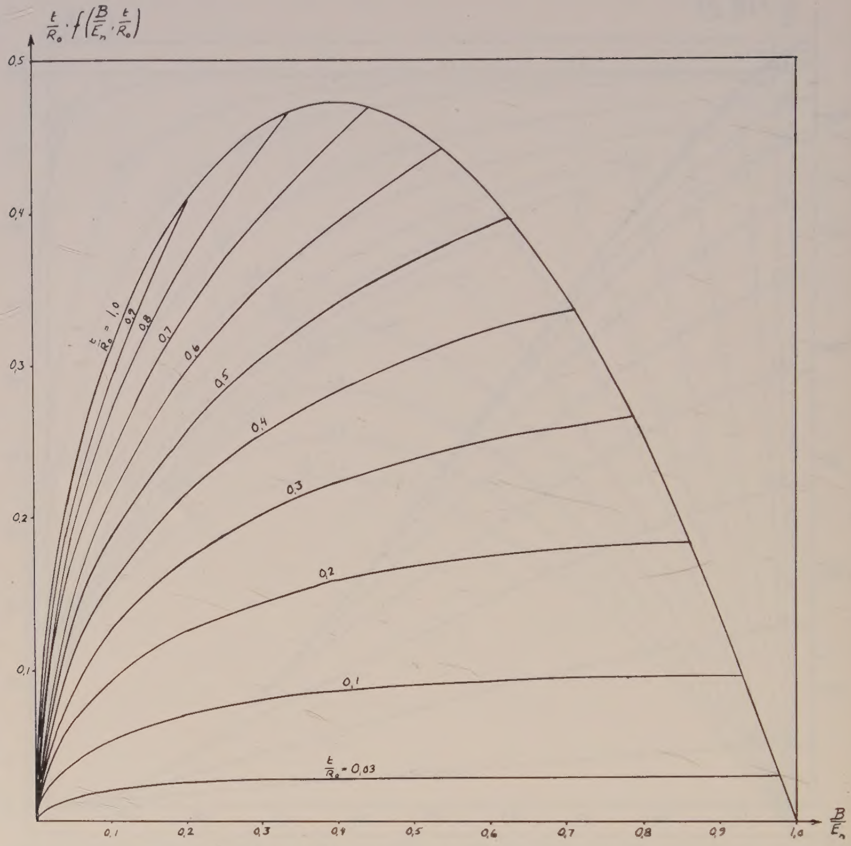

Fig. 6. Differential pulse-height distribution multiplied by the thickness of the radiator.

The differential sensitivity is then obtained by deriving with respect to B (e.g. implicitly over the parameter E').

The results are

$$A(E_n, B, t) = \begin{cases} \text{I)} \quad N\sigma t\left(1 - \dfrac{E'}{E_n} + \dfrac{1}{3}\,\dfrac{t}{R_0}\,\dfrac{E_n}{E'}\right) \\ \text{II)} \quad N\sigma R_0 \cdot \dfrac{1}{3}\left[1 - \left(\dfrac{B}{E_n}\right)^{3/2}\right]^2 \end{cases}$$

$$a(E_n, B, t) = \begin{cases} \text{I)} \quad \dfrac{N\sigma R_0}{E_n}\left[\left(\dfrac{B}{E_n}\right)^{1/2}\left(\dfrac{E'}{E_n}\right)^{3/2} - \left(\dfrac{B}{E_n}\right)^2\right] = \dfrac{N\sigma t}{E_n}\left(\dfrac{B}{E'}\right)^{1/2} \\ \text{II)} \quad \dfrac{N\sigma R_0}{E_n}\left[\left(\dfrac{B}{E_n}\right)^{1/2} - \left(\dfrac{B}{E_n}\right)^2\right]. \end{cases}$$

Table 1.

The Function $F\left(\frac{B}{E_n}, \frac{t}{R_0}\right)$.

Only the decimals are given. $F = 0.\text{xxxx}$.

$\frac{t}{R_0}$ \ $\frac{B}{E_n}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	1.0	9000	8000	7000	6000	5000	4000	3000	2000	1000	0
0.001	9789	8968	7984	6989	5992	4993	3994	2995	1996	0996	0
0.003	9635	8908	7951	6967	5975	4980	3983	2986	1988	0989	0
0.01	9333	8733	7845	6893	5918	4934	3945	2953	1959	0963	0
0.03	8845	8364	7583	6697	5764	4807	3838	2860	1877	0890	0
0.1	7892	7525	6891	6127	5288	4402	3487	2552	1603	0644	0
0.2	7019	6707	6158	5480	4717	3897	3037	2148	1237	0356	0
0.3	6349	6066	5564	4936	4224	3449	2630	1776	0899	0237	0
0.4	5784	5521	5050	4458	3781	3042	2254	1429	0674	0178	0
0.5	5286	5037	4590	4025	3377	2664	1903	1145	0539	0143	0
0.6	4836	4598	4169	3627	3001	2312	1592	0954	0450	0119	0
0.7	4422	4193	3780	3255	2649	1990	1364	0818	0385	0102	0
0.8	4037	3815	3415	2906	2325	1741	1194	0715	0337	0089	0
0.9	3675	3460	3070	2587	2067	1548	1061	0636	0300	0079	0
1.0	3333	3126	2764	2328	1860	1393	0955	0572	0270	0071	0

Table 2.

The Function $f\left(\frac{B}{E_n}, \frac{t}{R_0}\right)$.

Only the decimals are given. $F = 0.\text{xxxx}$.

$\frac{t}{R_0}$ \ $\frac{B}{E_n}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0—1	1	1	1	1	1	1	1	1	1	1—0
0.001	0	9696	9919	9963	9979	9987	9991	9993	9995	9996	0
0.003	0	9218	9767	9892	9939	9960	9972	9980	9984	9988	0
0.01	0	8192	9325	9666	9804	9872	9910	9933	9949	9959	0
0.03	0	6884	8486	9151	9470	9642	9743	9808	9851	9881	0
0.1	0	5389	7113	8077	8657	9024	9265	9430	9547	9632	0
0.2	0	4608	6238	7260	7946	8422	8762	9009	9193	6934	0
0.3	0	4192	5735	6751	7469	7992	8381	8676	8481	4623	0
0.4	0	3915	5389	6387	7113	7659	8077	8402	6361	3467	0
0.5	0	3772	5129	6106	6833	7390	7825	6933	5089	2774	0
0.6	0	3552	4923	5880	6602	7164	6910	5778	4240	2311	0
0.7	0	3422	4752	5691	6407	6530	5923	4952	3635	1981	0
0.8	0	3313	4608	5529	5906	5714	5182	4333	3180	1734	0
0.9	0	3219	4484	5086	5250	5079	4607	3852	2827	1541	0
1.0	0	3062	4072	4577	4725	4571	4146	3467	2544	1387	0

I) is used if the solution of $\frac{t}{R_0} = \left(\frac{E'}{E_n}\right)^2 - \left(\frac{B}{E_n}\right)^{3/2} \left(\frac{E'}{E_n}\right)^{1/2}$ gives $\frac{E'}{E_n} < 1$.

II) is used if the equation gives $\frac{E'}{E_n} > 1$.

It is convenient for the numerical calculation to write the sensitivities as follows

$$A\,(E_n,\, B,\, t) = N\,\sigma\, t \cdot F\left(\frac{B}{E_n},\, \frac{t}{R_0}\right),$$

$$a\,(E_n,\, B,\, t) = \frac{N\,\sigma\, t}{E_n} \cdot f\left(\frac{B}{E_n},\, \frac{t}{R_0}\right).$$

F and f have been calculated correct to four figures in tab. 1 and 2. F, f, $\frac{t}{R_0}\,F$, and $\frac{t}{R_0}\,f$ are found in Figs. 3–6.

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